

# OVERCOMING THE BACK REACTION ON TURBULENT MOTIONS IN THE PRESENCE OF MAGNETIC FIELDS

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## ABSTRACT

Standard magnetohydrodynamic theories, such as the mean field dynamo theory, have been criticized when the back reaction of the magnetic field on turbulent motions is neglected. For the dynamo, this back reaction has been argued to suppress the turbulent motions required for optimal mean field production. Here it is suggested that if the magnetic field is spatially intermittent, for example residing in flux tubes, the back reaction on turbulent flows may be significantly reduced.

*1. Introduction-* The Mean Field Dynamo (MFD), an elegant theoretical mechanism that allows a large scale magnetic field to grow exponentially at the expense of shear and small scale turbulent energy, has been studied to explain the origin of magnetic fields in astrophysical bodies such as planets, stars, and galaxies [1,2,3,4]. The difficulties with dynamo physics highlight fundamental issues of magneto-hydrodynamics that are still not well understood. In particular, the effect of magnetic fields on turbulent flows.

In principle, the MFD growth of a large scale magnetic field in a differentially rotating system occurs as turbulent motions induce formation of magnetic loops from a seed field [1,2,3]. As a result of the coriolis and buoyancy forces, the turbulence is cyclonic; the loops in both hemispheres twist in the same direction, creating a large scale loop of poloidal field. The turbulent diffusion of the outer portions of these mean field loops ensures that the net flux of mean field lying in the region of interest is non-vanishing [1]. Differential rotation shears the large scale poloidal field, generating a large scale toroidal field. The new field then incurs the same loop forming process, providing the feedback for exponential growth. The strength of the field is limited by the available turbulent energy. Numerical simulations of “kinematic” dynamos in which the back reaction of the field growth on the turbulent eddies is neglected, can produce magnetic topologies consistent with observations of stellar and galactic magnetic fields [5,6].

In reality, the small scale, root mean squared (rms) field energy density grows much more rapidly than the mean field [1], violating the kinematic approximation. Though, for example, observations of the solar photosphere and the dispersed heavy element distri-

bution in the Galaxy indicate the presence of reasonably uninhibited turbulent motions in amidst equipartition magnetic fields [1], theoretical studies [7,8] suggest that the back reaction should suppress these motions and thereby inhibit the dynamo. As first discussed by Piddington [9], the argument proceeds as follows: As an eddy tries to displace a parcel of fluid threaded by a field loop in equipartition with the turbulent energy density, the loop acts as a rubber band, restricting further motion, and recoiling the parcel to its origin. Statistically, transport of material is significantly reduced. Simulations support this intuition by finding that an increasingly large fraction of plasma motions are locked into oscillating modes rather than zero frequency (diffusive) modes as the magnitude of the initial ordered field is increased [8].

Here it is suggested that turbulent motion may survive the back reaction if the magnetic field is concentrated in flux tubes. Because the tubes' Alfvén speeds can be larger than the eddy velocities, reconnection between tubes can be rapid. This would reduce the back reaction force on the turbulent velocities. The diffusion and helicity coefficients of the MFD equation would be reduced from the kinematic theory only by the fraction of a typical tube thickness which does not reconnect in an eddy turnover time. (In the special case for which the magnetic field is totally composed of topologically unlinked loops or cells, interchange motions between the cells would not be restricted by magnetic forces and reconnection would not be required for diffusive motions.)

First, the derivation of the dynamo helicity and diffusion coefficients is outlined and the specific role of the kinematic approximation is highlighted. The stages of dynamo

growth are discussed and the formation and role of flux tubes is then addressed. An estimate of the dynamo coefficients is given based on the physical ideas presented. Finally, a similar role of intermittency for star forming regions is mentioned.

*2. Dynamo Coefficients and Interpretation of Approximations-* The magnetic field and velocity are taken to be  $\mathbf{B} = \mathbf{b} + \bar{\mathbf{B}}$  and  $\mathbf{V} = \mathbf{v} + \bar{\mathbf{V}}$  respectively, where  $\mathbf{b}$  and  $\mathbf{v}$  are fluctuating quantities with zero mean and  $\bar{\mathbf{B}}$  and  $\bar{\mathbf{V}}$  are mean quantities. The Reynolds relations [11] are also required. These are  $\partial_{t,\mathbf{x}}\langle K_i H_j \rangle = \langle \partial_{t,\mathbf{x}}(K_i H_j) \rangle$ , and  $\langle \bar{K}_i h_j \rangle = 0$ , where  $K_i = \bar{K}_i + k_i$  and  $H_j = \bar{H}_j + h_j$  are components of vector functions of position  $\mathbf{x}$  and time  $t$ , the brackets indicate the mean value, and  $\partial_{t,\mathbf{x}}$  is the derivative with respect to  $\mathbf{x}$  or  $t$ . For ensemble averages, these relations hold when the turbulence is correlated on time scales short relative to the variation time scales of the mean quantities. For the spatial average, defined by  $\langle \mathbf{K}(\mathbf{x}, t) \rangle = |\zeta|^{-3} \int_{\mathbf{x}-\zeta}^{\mathbf{x}+\zeta} \mathbf{K}(\mathbf{s}, t) d^3\mathbf{s}$ , these relations hold when the average is taken over a large enough scale, that is when  $l \ll |\zeta| \ll L$ , where  $L \sim \bar{B}/\nabla \bar{B}$  is the scale of the mean field variation, and  $l \sim b/\nabla b \sim v/\nabla v$ .

The induction equation derived from the non-relativistic Maxwell equations is [1]

$$\partial(\bar{\mathbf{B}} + \mathbf{b})/\partial t = \nabla \times [(\bar{\mathbf{V}} + \mathbf{v}) \times (\bar{\mathbf{B}} + \mathbf{b})] + \nu_M \nabla^2 (\bar{\mathbf{B}} + \mathbf{b}), \quad (1)$$

where  $\nu_M$  is the magnetic viscosity. Astrophysical magnetic Reynolds numbers are large and the last term in (1) is unimportant on the energy containing eddy scales for destroying magnetic field energy (but it does provide dissipation on the smaller scales allowing a turbulent cascade, and is important locally, at the intersections between thin flux tubes.)

Taking the average of (1), ignoring the last term, gives the MFD equation

$$\partial \bar{\mathbf{B}} / \partial t = \nabla \times [\bar{\mathbf{V}} \times \bar{\mathbf{B}} + \langle \mathbf{v} \times \mathbf{b} \rangle]. \quad (2)$$

The turbulent electromotive force (TEMF) given by  $\langle \mathbf{v} \times \mathbf{b} \rangle$ , is written [2]

$$\langle \mathbf{v} \times \mathbf{b} \rangle = \tilde{\alpha}_{ij} \bar{\mathbf{B}}_j + \tilde{\beta}_{ijk} \nabla_j \bar{\mathbf{B}}_k, \quad (3)$$

where  $\tilde{\alpha}_{ij}$  is the helicity dynamo coefficient, and  $\tilde{\beta}_{ijk}$  is the turbulent diffusion dynamo coefficient. Under the assumption of isotropic turbulence  $\tilde{\alpha}_{ij} = \delta_{ij} \tilde{\alpha}$  and  $\tilde{\beta}_{ijk} = \epsilon_{ijk} \tilde{\beta}$ .

Working in a frame for which  $\bar{\mathbf{V}} = 0$ , subtracting (2) from (1) gives

$$\partial \mathbf{b} / \partial t = \nabla \times (\mathbf{v} \times \bar{\mathbf{B}}) + \nabla \times [\mathbf{v} \times \mathbf{b} - \langle \mathbf{v} \times \mathbf{b} \rangle] + \mathbf{b} \cdot \nabla \bar{\mathbf{V}}. \quad (4)$$

Usually, for simplicity, the 3rd and 4th terms of (4) are neglected straight away, which is a procedure called the “first order smoothing (FOS) [1,2]” approximation. Instead, I will keep these terms and then show what this approximation means. Plugging (4) into (3), the dynamo coefficients become expansions of time-ordered exponential series [13] in powers of  $\tau_c / \tau_{ed}$ , where  $\tau_c$  is the turbulence correlation time and  $\tau_{ed} \sim l/v$  is the eddy turnover time. Under the assumption of isotropy, taking the first terms in the series give the standard forms [1,2]

$$\tilde{\alpha} = (-1/3) \langle \mathbf{v}(t) \cdot \int_0^t dt' \nabla \times \mathbf{v}(t') \rangle, \quad (5)$$

$$\tilde{\beta} = (1/3) \langle \mathbf{v}(t) \cdot \int_0^t dt' \mathbf{v}(t') \rangle. \quad (6)$$

Non-vanishing  $\tilde{\alpha}$  means non-vanishing helicity, essential for mean field growth as described above. That  $\tilde{\beta}$  acts as a diffusion coefficient for the mean field is evident from plugging (6)

into the TEMF in (2). For spatially homogeneous turbulence,  $\tilde{\beta}$  becomes the coefficient of the diffusion term on the right side.

Note that dropping the higher order terms that led to (5) and (6) is essentially the equivalent of the FOSA. The inclusion of higher order terms is an additional complication which is extraneous to, and independent of the focus on the back reaction in this paper: The back reaction reflects the specific effect of the magnetic field on the velocity, not the combination of velocities that appear in the dynamo coefficients.

In the usual kinematic approximation,  $\mathbf{v}$  is prescribed independently of  $\mathbf{b}$  and  $\bar{\mathbf{B}}$ . This motivates the use of (4) to eliminate  $\mathbf{b}$  in (3). However, the functional forms of the dynamo coefficients are exactly valid even when  $\mathbf{v} = \mathbf{v}(\mathbf{b}, \bar{\mathbf{B}})$ . The specific  $\mathbf{v}(\mathbf{b}, \bar{\mathbf{B}})$  will depend on the application, but the field would always be inhibitive to turbulent motions. Note that because the time scale for growth of the small scale field is much shorter than that of the mean field [1], the most important back reaction comes from the small scale field. This is the natural interpretation of studies [7] which show effects of the back reaction for values of the mean field much less than equipartition with the turbulence.

*3. Phases of Dynamo Growth and Flux Tube Formation-* Observations of the sun [14,15] and simulations of MHD turbulence in low  $\beta_{ave}$  ( $\equiv \bar{P}_{part}/\bar{P}_{mag}$  where  $\bar{P}_{part}$  and  $\bar{P}_{mag}$  are the average particle and magnetic pressures) plasmas [16] indicate that the magnetic field tends to concentrate in flux tubes. Intermittency in magnetic field strength in the Galactic interstellar medium is seen as well [17]. Determining the size of flux tubes and the role of  $\beta_{ave}$  will be addressed later but how such intermittent structures form is

addressed first.

A working mean field dynamo would incur phases given an initial seed field [13]. In the 1st phase, turbulent energy stirs up the rms random magnetic field to equipartition. In the 2nd phase, in principle, the mean field also nears equipartition with the turbulence and/or shear while the small scale field remains at or near equipartition. In the 3rd phase, the dynamo works to sustain the mean field. The back reaction is straightforwardly unimportant only during the 1st phase which lasts for a time  $\sim \tau_{ed}$ .

Flux tubes can form in phase 1 and Ref. [18] is relevant. There, the evolution of a seed magnetic field in homogeneous, isotropic turbulence is studied. An important result of [18] is that the field tends to concentrate locally into flux tubes or ropes. The field only grows in a local region when turbulence conspires to produce to proper stretching, twisting and folding [19]. Assuming that the mean field is constant over an eddy turnover time scale,  $d\mathbf{B}/dt \sim d\mathbf{b}/dt$  so that the induction equation for the total field can be used to explore the growth of the random field for phase 1. In Lagrangian form

$$dB_i/dt = B_i \nabla_i v_j. \quad (7)$$

As in [1], consider the initial  $B_i(0)$  to be aligned with a line segment  $\delta x_i(0)$ . Then at all later times  $\delta x_i(t)$  is aligned with  $B_i(t)$ . Eq. (8) then implies that the length of a line segment parallel to the field satisfies

$$dl(x, t)/dt = f(x, t)l(x, t), \quad (8)$$

where  $f(x, t)$  is a random function of the turbulent velocities. Although  $\langle l(x, t) \rangle$  can be shown to increase exponentially [1],  $l(x, t)$  is equally likely to decrease or increase at a

position  $x$ . Similarly, the field would be equally likely to increase or decrease there, and thus a natural spatial intermittency would result. The rms field in this picture, grows nearly to equipartition with the volume averaged turbulent flow energy density, which likely happens in only of order a time  $\sim \tau_{ed}$  [1] for a modest seed field. The back reaction is most inhibitive after equipartition ensues, and this is therefore the case of interest for this paper.

The growth of field in a particular localized region results from stretching and dragging of seed flux by a turbulent flow. As the bundle or tube is dragged favorably for exponential growth, material will be inhibited from seeping into the tube from the ends since there the field is weak and the force on particles opposes the direction of field line convergence [20]. The amount of mass in a tube should thus remain the same, or decrease. Assuming that it remains the same, incompressibility implies a constant volume. Thus the cross sectional area of the tube  $\pi r_t^2 \propto l^{-1}$ , and from flux freezing  $B_t \propto l$ . The edge of the tube is a current sheet, with no flow normal to it, and thus pressure is balanced across it :

$$\beta_t + 1 = (\beta_{ext} + 1)(B_{ext}/B_t)^2, \quad (9)$$

where  $\beta_t \equiv 8\pi P_{part,t}/B_t^2$  and  $\beta_{ext} \equiv 8\pi P_{part,ext}/B_{ext}^2$  with  $B_{ext}$  is of order the initial seed field,  $P_{part,t}$  is the tube particle pressure, and  $P_{part,ext}$  is the external particle pressure. Eq. (9) shows that  $\beta_t \ll \beta_{ext}$  since  $B_t \gg B_{ext}$  in equilibrium.

Since the tube pressure is balanced by the external pressure, the magnetic energy density can be higher than the turbulent energy density at the tube locations when  $\beta_t$  is small, and then the volume filling fraction of the tubes would necessarily be small. To



see this note that the average magnetic and particle pressures satisfy  $\bar{P}_{mag} \sim f P_{mag,t}$  and  $\bar{P}_{part} \sim (1-f) P_{part,ext}$  where  $f$  is the fraction of volume occupied by magnetic flux tubes,  $P_{mag,t}$  is the magnetic pressure in the flux tubes and  $P_{part,ext}$  is the particle pressure external to the tubes. Thus  $\beta_{ave} \sim [(1-f)/f] P_{part,ext}/P_{mag,t} = (1+\beta_t)(1-f)/f$ . Thus  $f/(1-f) = (1+\beta_t)/\beta_{ave}$ .

Each energy containing (outer) scale eddy of wavelength  $l$  stretches a tube to length  $l$  and radius  $r_t$ . The thickness of each tube,  $r_t$ , can be estimated by balancing the magnetic and turbulent eddy drag forces [9,21]. This gives

$$(B_t^2/4\pi r_c)(\pi r_t^2) \sim C_d \rho_{ext} v_l^2 2r_t, \quad (10)$$

where  $B_t$  is the magnitude of the field in the flux tube,  $\rho_{ext}$  is the density outside the flux tube, and  $C_d$  is the coefficient of turbulent drag. Since  $l$  is a wavelength, the radius of curvature  $r_c$  can be estimated by  $l/4$  when the tube maximally responds to the turbulence. In equilibrium,  $B_t^2/8\pi \sim P_{part,ext}(1+\beta_t)$ , so that (10) gives, for  $(1-f) \sim 1$ ,

$$r_t/l = C_d \Gamma M_l^2 / 4\pi \sim 0.06 M_l^2 (1+\beta_t), \quad (11)$$

where  $\Gamma$  is the adiabatic index and  $M_l^2 \equiv v_l^2/(\Gamma P_{ext}/\rho_{ext}) \sim \beta_{ave}^{-1} \sim f(1+\beta_t)^{-1}$ , when equipartition between turbulent and magnetic energy is assumed. For the last similarity in (11),  $C_d$  was estimated from the “drag” crisis [21] which reduces  $C_d < 1$  at large turbulent Reynolds number  $R_l$ . Assuming  $R_l \gtrsim 1000$ ,  $C_d \sim 0.4$ .

*4. Role of Flux Tubes-* Once dynamo growth enters the 2nd and 3rd phases, the back reaction of the field on the turbulent eddy becomes important with respect to the net

transport of magnetic field. In particular, the rapid growth in magnetic tension inhibits it from traveling much more than  $\sim l$ . Unless reconnection with another tube can happen before the tension response, the tube will react back. Both the diffusion and helicity coefficients require inhibited motions of the turbulent velocity. For example, inhibited turbulent diffusion would mean that an ink mark on some tube statistically incurs zero net displacement from its initial location (i.e. oscillating motions) instead of increasing its separation from the initial point with time.

No inhibition of motions would mean that a tube could reconnect with a partner in the time it takes to move a distance  $r_t$ , namely a time  $r_t/v_l$ . After a reconnection, the tube would change one of its ends, and would then move in a different random direction from whence it came, before impacting another tube. The process would continue, enabling for example, an effective diffusion. The reconnection time scale is  $r_t F(R_{M,t})/v_{A,t}$  where  $F(R_{M,t})$  is the function of the magnetic Reynolds number associated with the length scale  $r_t$ , and  $v_{A,t}$  is the tube Alfvén speed. There would be no inhibition to turbulent motions when

$$v_{A,t}/v_l \sim M_l^{-1}(1 + \beta_t)^{-1/2} \sim \beta_{ave}^{1/2}/(1 + \beta_t)^{1/2} > F(R_{M,t}), \quad (12)$$

where the second similarity follows from equipartition. For Sweet-Parker (SP) reconnection,  $F(R_{M,t}) \sim R_{M,t}^{1/2}$  while for Petschek (PK) reconnection  $F(R_{M,t}) \sim \text{Ln} R_{M,t}$  [1]. Note that  $R_{M,t} \sim (r_t/l)R_{M,l} = 0.06R_{M,l}M_l^2(1 + \beta_t) \sim 0.06R_{M,l}(1 + \beta_t)/\beta_{ave}$  from (11), where  $R_{M,l}$  is the standard magnetic Reynolds number associated with  $l$ . For the more stringent SP case, (12) then becomes  $\beta_{ave}/(1 + \beta_t) > 0.25R_{M,l}^{1/2}$  which is likely satisfied in or above the solar convection zone of the sun [1]. Note that if the field were diffuse and not

concentrated in flux tubes, then then  $\beta_{ave} \sim \beta_t$  and the inequality in (12) could not be satisfied.

Note that (11), and the line following it, imply that  $r_t/L \lesssim f$ . But each tube fills a fraction  $\sim r_t^2/L^2 \lesssim f^2$  of an eddy volume. Thus there are  $\gtrsim 1/f$  tubes per eddy volume when  $(1 - f) \sim 1$ .

5. *Application to the Dynamo Coefficients-* The simplest way to describe the effect of flux tubes, is to say that fast reconnection significantly reduces the back reaction terms of the Lorentz force on the velocity field in the equation of motion. Thus the velocities in the dynamo coefficients would be approach their kinematic values the more efficient the reconnection. In the presence of equipartition magnetic flux tubes, the reduction from their kinematic limit would be determined by the fraction of tube Lorentz force that contributes to the back reaction, namely the fraction that cannot reconnect during  $\tau_{ed}$ . Equivalently, using (13), and noting that dynamo coefficients depend on two powers of the velocity, we have

$$\tilde{\alpha}_{br} \sim \text{Min} [\tilde{\alpha}_{kin}, \beta_{ave}(1 + \beta_t)^{-1} \tilde{\alpha}_{kin}/F(R_{M,t})^2], \quad (13)$$

and

$$\tilde{\beta}_{br} \sim \text{Min} [\tilde{\beta}_{kin}, \beta_{ave}(1 + \beta_t)^{-1} \tilde{\beta}_{kin}/F(R_{M,t})^2], \quad (14)$$

where the subscripts *kin* and *br* refer to the kinematic values and the values including the back reaction, respectively. The right sides of (13) and (14) are the minima of the quantities in brackets.

It can be useful to think of the diffusion coefficient  $\tilde{\beta}_{br}$ , as measuring the fraction of

eddy energy per mass, contained in motions which random walk rather than oscillate. For a given amount of total eddy energy, a stronger back reaction means a higher fraction of non-zero frequency modes [8]. To see this, note that when the velocity is given by a stationary random field,  $\langle \mathbf{v}(t) \cdot \mathbf{v}(t + \tau) \rangle = C(\tau)$ . Then the Fourier transform gives  $C(\omega) = \int_{-\infty}^{\infty} d\tau \text{Exp}[i\omega\tau] \langle \mathbf{v}(t) \cdot \mathbf{v}(t + \tau) \rangle$ . The zero frequency component, by comparison with (6) then satisfies  $\tilde{\beta} = (1/4)\tilde{C}(0)$ . where  $\tilde{C}(0)$  is the Fourier transform of the velocity correlation (i.e. the power spectrum of the velocity field) evaluated at zero frequency. The amount of eddy energy per mass in the zero frequency modes (i.e. the fraction contributing to the diffusion coefficient) is the non-zero contribution to the diffusion coefficient

*6. Discussion* Rapid reconnection, resulting from a concentration of magnetic fields into low  $\beta_t$  regions, may overcome the back reaction on turbulent motions. Diffusion of the mean field and helicity would be enabled not necessarily by removing a large amount of field energy on the outer scale, but by allowing individual flux tubes to avoid recoiling back to their points of origin. The most important motions would be enabled on the energetically dominant scales. However, a steady forcing of the turbulence on these outer scales would give a cascade to small scales as in (e.g.) [22], maintaining a constant magnetic + turbulent energy density on the outer scale. The irreversible dissipation would occur on the smallest scales. Eqns (13) and (14) apply most effectively when  $\beta_{ave} \gg \text{Max}[1, \beta_t]$ . The value of  $\beta_t$  determines how effectively flux tubes evacuate and there may be a non-linear dependence on  $\beta_{ave}$ . Actual values will have to await future simulations.

If the shear energy were much larger than the turbulent energy and could be dumped

into the field fast enough, the magnetic energy may exceed the turbulent energy. Then the first similarity in the line following (11) would become  $\lesssim$  and the inequality condition between the last terms in (12) would be stricter than required, since the third term would be  $\lesssim$  the first two.

Finally, note that a similar rapid reconnection between evacuated tubes in low  $\beta_{ave}$  star forming molecular cloud regions of the ISM may remove material from field lines and initiate collapse [23]. In an initially uniform  $\beta_{ave} \ll 1$  plasma, non-linear compressional Alfvén waves clump material on the scale of the energy containing eddies to density enhancements of order  $1/\beta_{ave}$ . The Alfvén speed associated with the sparse regions is large, allowing rapid reconnection, closed loops formation, and dissipation. Thus intermittency can also lead to fast reconnection in plasmas with  $\beta_{ave} \ll 1$ .

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